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Conformal anomalies of CFT's with boundaries

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Abstract: The trace anomaly of conformal field theories in four dimensions is characterized by 'a' and 'c'-functions. The scaling properties of the effective action of a CFT in the presence of boundaries is shown to be determined by a, c and two new functions (charges) related to boundary effects. The boundary charges are computed for different theories and different boundary conditions. One of the boundary charges depends on the bulk c charge.

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Contents

1 Anomaly of the effective action

Conformal anomalies are well-known. In a conformal field theory (CFT) in four dimensions the expectation value of the trace of stress energy tensor can be written in the following universal form [1]:

$$\langle T^{\mu}_{\mu} \rangle = -2a\ E - c\ I - \frac{c}{24\pi^2}\ \nabla^2 R\,, \tag{1.1}$$

$$E = \frac{1}{32\pi^2}\left[R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}\right], \tag{1.2}$$

$$I = -\frac{1}{16\pi^2}C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}\,, \tag{1.3}$$

where E is the volume density of the Euler characteristics of the underlying background manifold $\mathcal{M}$, and $C_{\mu\nu\lambda\rho}$ is the Weyl tensor of $\mathcal{M}$. Coefficient at $\nabla^2 R$ in (1.1) can be changed by adding a local counterterm. Coefficients at other terms in the anomaly are scheme independent.

One can also define the integral anomaly by the variation of the effective action W,

$$\mathcal{A} \equiv \partial_\sigma W[e^{2\sigma} g_{\mu\nu}]_{\sigma=0} = \int_{\mathcal{M}} \sqrt{g} d^4x \langle T^{\mu}_{\mu} \rangle\,, \tag{1.4}$$

under scaling with a constant factor σ. The right hand side of (1.4) relates $\mathcal{A}$ to the trace anomaly (1.1) and holds on a closed manifold.

By analogy with two-dimensional CFT's constants a and c are called charges, or a-function and c-function, when they are allowed to run under renormalization. There are arguments [2] that a-function changes monotonically when going from a critical point to a critical point.

The aim of this work is to study integral anomaly of the effective action (1.4) when $\mathcal{M}$ has a boundary. In this case the right hand side of (1.4) cannot be written solely as the

Theory	a	c	q_1	q_2	boundary condition
real scalar	$\frac{1}{360}$	$\frac{1}{120}$	$\frac{1}{15}$	$\frac{2}{35}$	Dirichlet
real scalar	$\frac{1}{360}$	$\frac{1}{120}$	$\frac{1}{15}$	$\frac{2}{45}$	Robin
Dirac spinor	$\frac{11}{360}$	$\frac{1}{20}$	$\frac{2}{5}$	$\frac{2}{7}$	mixed
gauge Boson	$\frac{31}{180}$	$\frac{1}{10}$	$\frac{12}{15}$	$\frac{16}{35}$	absolute
gauge Boson	$\frac{31}{180}$	$\frac{1}{10}$	$\frac{12}{15}$	$\frac{16}{35}$	relative

Table 1. Charges in the anomaly of the effective action.

integral of the trace anomaly since new boundary terms may appear. An experience with two dimensional CFT's shows that boundary terms in some quantum quantities, such as a g-function in the entropy, may be interesting from the point of view of the renormalization group [3, 4].

We study different CFT's in four dimensions with the boundary conditions which do not break the conformal invariance on the classical level and show that the integral anomaly has the following universal structure:

$$\mathcal{A} = -2a\ \chi_4 - c\ i + q_1 j_1 + q_2 j_2\,. \tag{1.5}$$

Quantities χ_4, i, j_1 and j_2 are scale invariant functionals, χ_4 is the Euler characteristics of the background manifold, i is the integral of I over $\mathcal{M}$, see (1.3). The boundaries result in two new terms with constants q_1 and q_2 (the boundary charges). The boundary functionals in (1.5) are

$$j_1 = \frac{1}{16\pi^2}\int_{\partial\mathcal{M}} \sqrt{H} d^3x\ C_{\mu\nu\lambda\rho} N^\nu N^\rho \hat{K}^{\mu\lambda} \equiv \frac{1}{16\pi^2}\int_{\partial\mathcal{M}} \sqrt{H} d^3x G_1\,, \tag{1.6}$$

$$j_2 = \frac{1}{16\pi^2}\int_{\partial\mathcal{M}} \sqrt{H} d^3x\ \mathrm{Tr}(\hat{K}^3) \equiv \frac{1}{16\pi^2}\int_{\partial\mathcal{M}} \sqrt{H} d^3x G_2\,. \tag{1.7}$$

We use the following notations: $g_{\mu\nu}$ is the metric of the background manifold $\mathcal{M}$, the metric induced on the boundary $\partial\mathcal{M}$ of $\mathcal{M}$ is $H_{\mu\nu} = g_{\mu\nu} - N_\mu N_\nu$, N^ν is a unit outward pointing normal vector to $\partial\mathcal{M}$, $K_{\mu\nu} = H^\lambda_\mu H^\rho_\nu N_{\lambda;\rho}$ is the extrinsic curvature tensor of $\partial\mathcal{M}$, $\hat{K}_{\mu\nu} = K_{\mu\nu} - H_{\mu\nu}K/3$ is a traceless part of $K_{\mu\nu}$. Conformal invariance of j_1, j_2 follows from the fact that $\hat{K}_{\mu\nu}$ transforms homogeneously under conformal transformations. Definitions of the Riemann tensor, the Ricci tensor, the scalar curvature, respectively, are $R^\lambda_{\mu\nu\rho} = -\Gamma^\lambda_{\mu\nu,\rho} + \ldots$, $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$, $R = R^\mu_\mu$. Our definitions of R and $K_{\mu\nu}$ coincide with those used by Dowker and Schofield in [5] and by Vassilevich in [6]. The only difference with those works is in the sign of $R^\lambda_{\mu\nu\rho}$ and direction of N^μ. In our paper, in [5], and in [6] the scalar curvature on S^n is positive. The curvatures constructed with the metric of $\partial\mathcal{M}$ will be denoted as $\hat{R}^\lambda_{\mu\nu\rho}$, $\hat{R}_{\mu\nu}$, $\hat{R}$.

Our results for the boundary charges in (1.5) are listed in table 1. Coefficient q_2 can depend on the type of the boundary conditions.

It should be noted that (1.5) for a scalar field with the Dirichlet boundary condition follows from results by Dowker and Schofield [5]. Table 1 agrees with [5] for this case.

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Functional j_2 was introduced for the first time in [7]. Anomalous rescalings of the effective action of Dirac fields and gauge bosons have been also studied by Moss and Poletti [8] for Einstein spaces with boundaries.

To our knowledge the model independent nature of eq. (1.5) has not been emphasized so far. It is important to note that eq. (2.1), which determines (1.5), can be derived in different regularization prescriptions, e.g. in the zeta-function method and the proper-time cutoff regularization, see [9]. Also, there are no local counterterms which can be added to the effective action to change the boundary terms in the integral anomaly. In this sense q_1 and q_2 are scheme independent.

In section 2 we relate the anomaly to known computations of boundary terms in the corresponding heat kernel coefficients. Anomalies in the models presented in table 1 are discussed in section 3. Concluding remarks are given in section 4. We point out a universal relation between c and q_1 (valid at least for all above models). It hints that q_1 is not an independent new charge.

2 The heat coefficient

We use the relation between the anomaly and the heat coefficient of a Laplace operator $\Delta = -\nabla^2 + X$ for the corresponding conformal theory

$$\mathcal{A} = \eta A_4 \,, \tag{2.1}$$

where $\eta = +1$ for Bosons and $\eta = -1$ for Fermions. We ignore in (2.1) a possible contribution of zero modes of Δ. The heat coefficients for the asymptotic expansion of the heat kernel of Δ are defined as

$$K(\Delta;t) = \mathrm{Tr}\; e^{-t\Delta} \simeq \sum_{p=0} A_p(\Delta)\; t^{(p-4)/2} \,, \quad t \to 0 \,. \tag{2.2}$$

If the classical theory is scale invariant the heat coefficient A_4 is a conformal invariant, see e.g. [9]. Therefore A_4 can be represented as a linear combination of conformal invariants constructed of the geometrical characteristics of $\mathcal{M}$, $\partial\mathcal{M}$ and embedding of $\partial\mathcal{M}$ in $\mathcal{M}$. On dimensional grounds, these invariants (in four dimensions) are χ_4, i, j_1 and j_2. The bulk part of A_4 is determined by χ_4 and i, and it is well known.

We are interested in boundary terms in A_4. To find them one should take into account the boundary term in χ_4. The definition of the Euler number for a four-dimensional manifold with a boundary is as follows:

$$\chi_4 = B_4[\mathcal{M}] + S_4[\partial\mathcal{M}] \,, \tag{2.3}$$

$$B_4[\mathcal{M}] = \int_{\mathcal{M}} \sqrt{g} d^4x\; E \,, \tag{2.4}$$

$$S_4[\partial\mathcal{M}] = \frac{1}{32\pi^2} \int_{\partial\mathcal{M}} \sqrt{H} d^3x Q \,, \tag{2.5}$$

$$Q = -8 \left[\det K_{\mu\nu} + \hat{G}^{\mu\nu} K_{\mu\nu} \right] \,, \quad \hat{G}^{\mu\nu} = \hat{R}^{\mu\nu} - \frac{1}{2} H^{\mu\nu} \hat{R} \,. \tag{2.6}$$

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Derivation of these formulae can be found in [5]. Some values of χ_4 are: $\chi_4 = 1$, if $\mathcal{M}$ is a domain in R^4 with the spherical boundary $\partial\mathcal{M} = S^3$; $\chi_4 = 0$, if $\mathcal{M}$ is a domain of a torus $S^1 \times R^3$ with a boundary $\partial\mathcal{M} = S^1 \times S^2$.

The boundary part of A_4, therefore, is

$$A_4^{\mathrm{bd}} = \eta(q_1 j_1 + q_2 j_2 - 2aS_4)\,. \tag{2.7}$$

It is the aim of computations to check that the coefficient at S_4 does equal $-2\eta a$, where a is the same constant which appears in the trace anomaly (1.1).

Appearance of S_4 among counter terms in a one-loop effective action dates back to works in 1980's, see [10]. If $\mathcal{M}$ is a domain of a flat Euclidean background and $\partial\mathcal{M} = S^3$ one easily finds for (1.5) that $\mathcal{A} = -2a$. In this case the anomaly is solely determined by S_4. Mode-by-mode computations of the anomaly for the spinor and gauge fields have been done in [11, 12].

Our starting point is formula (5.33) from Vassilevich's review [6] for A_4 for mixed boundary conditions. We put there $f = 1$, $n = 4$. The boundary conditions are

$$(\nabla_N - S)\Pi_+\phi = 0\,, \quad \Pi_-\phi = 0\,, \tag{2.8}$$

where $\nabla_N = N^\mu\nabla_\mu$, $\Pi_\pm$ are corresponding projectors, $\Pi_+ + \Pi_- = 1$, definition of S coincides with [6]. The Dirichlet or Robin conditions follow from (2.8) when $\Pi_+ = 0$ or $\Pi_- = 0$, respectively.

After converting total derivatives in the bulk into surface terms and some algebra the boundary part A_4^{bd} of A_4 can be written as

$$A_4^{\mathrm{bd}} = \frac{1}{(4\pi)^2}\int_{\partial\mathcal{M}} \sqrt{H} d^3x \ \mathrm{Tr}\ C_4\,, \tag{2.9}$$

$$C_4 = \Pi_+ C_4^+ + \Pi_- C_4^- + C_4^{+-}\,, \tag{2.10}$$

$$\begin{aligned} C_4^+ = &-\frac{1}{360}Q + \frac{1}{15}G_1 + \frac{2}{45}G_2 - \frac{1}{3}\left(X - \frac{1}{6}R\right)K + \frac{1}{2}\nabla_N\left(X - \frac{1}{6}R\right) \\ &+ \frac{4}{3}\left(S\Pi_+ + \frac{1}{3}K\right)^3 - 2\left(X - \frac{1}{6}R\right)S + \left(S + \frac{1}{3}K\right)\left(\frac{2}{15}\mathrm{Tr}K^2 - \frac{2}{45}K^2\right)\,, \end{aligned} \tag{2.11}$$

$$C_4^- = -\frac{1}{360}Q + \frac{1}{15}G_1 + \frac{2}{35}G_2 - \frac{1}{3}\left(X - \frac{1}{6}R\right)K - \frac{1}{2}\nabla_N\left(X - \frac{1}{6}R\right)\,, \tag{2.12}$$

$$\begin{aligned} C_4^{+-} = &-\frac{1}{3}(\Pi_+ - \Pi_-)\Pi_{+:a}\Omega_{a\mu}N^\mu \\ &- \frac{2}{15}\Pi_{+:a}\Pi_{+:a}K - \frac{4}{15}\Pi_{+:a}\Pi_{+:b}K^{ab} - \frac{4}{3}\Pi_{+:a}\Pi_{+:a}\Pi_+ S\,. \end{aligned} \tag{2.13}$$

Here we use 'flat' indices a, b in the tangent space to the boundary, see [6]. $\Omega_{\mu\nu}$ is the field strength of the connection defined by (2.10) in [6]. The notations are $\mathrm{Tr}K^m = K^{\mu_1}_{\mu_2}\dots K^{\mu_p}_{\mu_1}$, $K = \mathrm{Tr}K$.

In deriving (2.10)–(2.13) we used Gauss-Codazzi identities and the relations:

$$G_1 = R_{\mu\nu\lambda\rho}K^{\mu\lambda}N^{\nu}N^{\rho} - \frac{1}{2}R_{\mu\nu}(N^{\mu}N^{\nu}K + K^{\mu\nu}) + \frac{1}{6}KR\,, \tag{2.14}$$

$$G_2 = \mathrm{Tr}K^3 - K\mathrm{Tr}K^2 + \frac{2}{9}K^3\,, \tag{2.15}$$

$$\begin{aligned} Q = {} & 8R_{\mu\nu\lambda\rho}K^{\mu\lambda}N^{\nu}N^{\rho} - 8R_{\mu\nu}(N^{\mu}N^{\nu}K + K^{\mu\nu}) + 4KR \\ & + \frac{8}{3}K^3 + \frac{16}{3}\mathrm{Tr}K^3 - 8K\mathrm{Tr}K^2\,, \end{aligned} \tag{2.16}$$

which can be easily obtained from definitions (1.6), (1.7), (2.6).

Representation of the boundary terms in form (2.10)–(2.13) follows Moss and Poletti [8]. Calculations of A_4 in case of boundaries have been done by several authors. The key paper is by Branson and Gilkey [13]. A complete list of references can be found in [6].

3 Computations

3.1 Conformal scalar field

In this case $X = 1/6$. For the Dirichlet condition $\Pi_+ = 0$, $C_4^{+-} = 0$, and the boundary charges are determined by C_4^-, (2.12).

Conformally invariant Robin condition requires $S = -K/3$, $\Pi_+ = 1$. Then again $C_4^{+-} = 0$, and boundary charges follow from (2.11).

3.2 Massless Dirac spinor

In case of a massless Dirac field ψ the operator is $\Delta^{(1/2)} = (i\gamma^{\mu}\nabla_{\mu})^2$. The boundary conditions are mixed ones,

$$\Pi_-\psi\mid_{\partial\mathcal{M}} = 0\,, \quad (\nabla_N + K/2)\Pi_+\psi\mid_{\partial\mathcal{M}} = 0\,, \tag{3.1}$$

where $\Pi_\pm = \frac{1}{2}(1 \pm i\gamma_* N^{\mu}\gamma_{\mu})$, and γ_* is a chirality gamma matrix. Therefore, $X = R/4$, $S = -\Pi_+ K/2$.

The physical meaning of (3.1) is that the normal component of the spinor current vanishes on the boundary. Condition (3.1) does not break conformal invariance. The strength of the spin connection is

$$\Omega_{\mu\nu} = \frac{1}{4}R_{\mu\nu\sigma\rho}\gamma^{\sigma}\gamma^{\rho}\,.$$

The rest computation is straightforward. One finds

$$\mathrm{Tr}(\Pi_+C_4^+ + \Pi_-C_4^-) = r\left(-\frac{1}{360}Q + \frac{1}{15}G_1 + \frac{1}{315}G_2 + \frac{1}{72}KR + \frac{1}{1620}K^3 - \frac{1}{90}K\mathrm{Tr}(K^2)\right), \tag{3.2}$$

$$\mathrm{Tr}(C_4^{+-}) = r\left(-\frac{1}{12}R_{\mu\nu\lambda\rho}K^{\mu\lambda}N^{\nu}N^{\rho} - \frac{1}{15}\mathrm{Tr}(K^3) + \frac{1}{20}K\mathrm{Tr}(K^2)\right), \tag{3.3}$$

where $r = 4$ is the number of components of the Dirac spinor in four dimensions. In computing C_4^{+-} one uses the relation

$$\Pi_{+:a} = \frac{i}{2}\gamma^b\gamma_* K_{ba}\,.$$

The data of table 1 follow from the sum of (3.2) and (3.3) and relations (2.14)–(2.16).

3.3 Gauge boson

By following [6] we consider the quantization of an Abelian gauge field V_μ in the Lorentz gauge $\nabla V = 0$. The results of table 1 are valid for a gauge invariant combination

$$\mathcal{A} = A_4(\Delta^{(1)}) - 2A_4(\Delta^{(\mathrm{gh})})\,, \tag{3.4}$$

where $(\Delta^{(1)})^\nu_\mu = -\nabla^2\delta^\nu_\mu + R^\nu_\mu$ is the vector Laplacian, and $\Delta^{(\mathrm{gh})} = -\nabla^2$ is the Laplacian for ghosts.

We study two sorts of boundary conditions: the absolute (or electric [8]) boundary condition

$$N^\mu F_{\mu\nu}\,|_{\partial\mathcal{M}} = 0\,, \tag{3.5}$$

and relative (or magnetic [8]) boundary condition

$$N^\mu \tilde{F}_{\mu\nu}\,|_{\partial\mathcal{M}} = 0\,, \tag{3.6}$$

where $F_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu$, and $\tilde{F}_{\mu\nu}$ is the Hodge dual of $F_{\mu\nu}$.

The both conditions are manifestly gauge and conformally invariant.

Condition (3.5) requires that components of an electric field normal to $\partial\mathcal{M}$ and components of the magnetic field which are tangential to $\partial\mathcal{M}$ vanish on the boundary. This means that the boundary is a perfect conductor. In condition (3.6) the roles of electric and magnetic fields are interchanged.

In the Lorentz gauge the absolute boundary condition is reduced to the following conditions on the vector field and ghosts [6]:

$$(\delta^\nu_\mu\nabla_N + K^\nu_\mu)V^+_\nu\,|_{\partial\mathcal{M}} = 0\,, \quad V^-_\mu\,|_{\partial\mathcal{M}} = 0\,, \tag{3.7}$$

where $V^\pm = \Pi_\pm V$, and

$$(\Pi_+)^\nu_\mu = \delta^\nu_\mu - N_\mu N^\nu\,, \quad (\Pi_-)^\nu_\mu = N_\mu N^\nu\,. \tag{3.8}$$

The corresponding boundary condition for a ghost field c is

$$\partial_N c\,|_{\partial\mathcal{M}} = 0\,. \tag{3.9}$$

The relative condition in the same gauge is

$$(\nabla_N + K)V^+_\mu\,|_{\partial\mathcal{M}} = 0\,, \qquad V^-_\mu\,|_{\partial\mathcal{M}} = 0\,, \tag{3.10}$$

$$(\Pi_+)^\nu_\mu = N_\mu N^\nu\,, \qquad (\Pi_-)^\nu_\mu = \delta^\nu_\mu - N_\mu N^\nu\,. \tag{3.11}$$

$$c\,|_{\partial\mathcal{M}} = 0\,. \tag{3.12}$$

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By taking into account that $X^\mu_\nu = R^\mu_\nu$, $(\Omega_{\lambda\rho})^\nu_\mu = R^\nu_{\mu\lambda\rho}$, for the vector component with the absolute condition (3.7), (3.9) ($S^\mu_\nu = -K^\mu_\nu$) we find

$$\begin{aligned}\mathrm{Tr}(\Pi_+ C_4^+ + \Pi_- C_4^-) &= (N^\mu R_{\mu a})^{:a} - \frac{1}{6}R_{,N} \\ &\quad - \frac{4}{360}Q + \frac{4}{15}G_1 - \frac{8}{7}G_2 - \frac{4}{9}KR + R_{\mu\nu}(N^\mu N^\nu K + K^{\mu\nu})\,, \end{aligned} \tag{3.13}$$

$$\mathrm{Tr}(C_4^{+-}) = \frac{2}{3}R_{\mu\nu\lambda\rho}K^{\mu\lambda}N^\nu N^\rho + \frac{4}{5}\mathrm{Tr}(K^3) - \frac{4}{15}K\mathrm{Tr}(K^2)\,. \tag{3.14}$$

For the ghost part with condition (3.9) $S = 0$ and

$$C_4 = C_4^+ = -\frac{1}{12}R_{,N} - \frac{1}{360}Q + \frac{1}{15}G_1 + \frac{2}{45}G_2 + \frac{1}{18}KR + \frac{14}{405}K^3 + \frac{2}{45}K\mathrm{Tr}(K^2)\,, \tag{3.15}$$

For the vector component with relative conditions (3.10), (3.11) ($S^\mu_\nu = -K\delta^\mu_\nu$)

$$\begin{aligned}\mathrm{Tr}(\Pi_+ C_4^+ + \Pi_- C_4^-) &= -(N^\mu R_{\mu a})^{:a} + \frac{1}{6}R_{,N} - \frac{4}{360}Q + \frac{4}{15}G_1 + \frac{8}{45}G_2 \\ &\quad - \frac{4}{9}KR + R_{\mu\nu}(N^\mu N^\nu K + K^{\mu\nu}) - \frac{34}{405}K^3 - \frac{4}{45}K\mathrm{Tr}(K^2)\,, \end{aligned} \tag{3.16}$$

$$\mathrm{Tr}(C_4^{+-}) = -\frac{2}{3}R_{\mu\nu\lambda\rho}K^{\mu\lambda}N^\nu N^\rho - \frac{8}{15}\mathrm{Tr}(K^3) + \frac{16}{15}K\mathrm{Tr}(K^2)\,. \tag{3.17}$$

For the ghost part with condition (3.12)

$$C_4 = C_4^- = \frac{1}{12}R_{,N} - \frac{1}{360}Q + \frac{1}{15}G_1 + \frac{2}{35}G_2 + \frac{1}{18}KR\,. \tag{3.18}$$

In deriving (3.13), (3.16) we used the identity

$$\frac{1}{2}R_{,N} = (N^\mu R_{\mu a})^{:a} + KR_{\mu\nu}N^\mu N^\nu - R_{\mu\nu}K^{\mu\nu} + N^\mu N^\nu N^\lambda R_{\mu\nu;\lambda}\,.$$

The first term in the right hand side of this identity is the total derivative on $\partial\mathcal{M}$. Let us emphasize that the two sets of boundary conditions for the gauge field result in the same boundary terms in the anomalous scaling of the effective action.

4 Concluding remarks

The aim of this paper was to demonstrate a model independent form of the integral anomaly (1.5) in the presence of boundaries and obtain specific values of the boundary charges for some CFT's. It would be important now to study evolution of the boundary charges under the renormalization group.

Computations of boundary charges for other models can be continued along the lines of the present paper. Extensions of (1.5) to higher dimensional CFT's, say to 6 dimensions, are possible. Since the number of scale invariant structures is increasing we expect more boundary charges. A principal challenge here is the knowledge of boundary terms in the heat coefficient A_6.

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An interesting issue is a possible relation between bulk charges a, c and boundary charges q_1, q_2. A conjecture of [14] is that q_1 and c may be related. Indeed, all models presented in table 1 satisfy a universal relation

$$q_1 = 8c\,. \tag{4.1}$$

We leave a general proof and implications of (4.1) for a future analysis. Some arguments in the favour of (4.1) are presented in [14].

It should be noted that formula (1.5) was also discussed in a recent work [15] which appeared several days earlier of the present publication.

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